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Viscoelastic fluid flow and heat transfer over a stretching sheet under the effects of a non-uniform heat source, viscous dissipation and thermal radiation

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Abstract

The problem of flow and heat transfer of an incompressible homogeneous second grade fluid over a non-isothermal stretching sheet in the presence of non-uniform internal heat generation/absorption is investigated. The governing partial differential equations are converted into ordinary differential equations by a similarity transformation. The effects of viscous dissipation, work due to deformation, internal heat generation/absorption and thermal radiation are considered in the energy equation and the variations of dimensionless surface temperature as well as the heat transfer characteristics with various values of non-dimensional viscoelastic parameter k_1, Prandtl number σ, Eckert number $E_c(E_c')$, radiation parameter N_R, and the coefficients of space-dependent (A^*) and temperature-dependent (B^*) internal heat generation/absorption are graphed and tabulated. Two cases are studied, namely, (i) the sheet with prescribed surface temperature (PST case) and (ii) the sheet with prescribed heat flux (PHF case).

Keywords: Flow and heat transfer; Viscoelastic fluid; Frictional heating; Non-uniform heat source; Elastic deformation; Radiation; Stretching sheet

1. Introduction

Boundary layer behaviour over a moving continuous solid surface is an important type of flow occurring in a number of engineering processes. To be more specific, heat-treated materials travelling between a feed roll and a wind-up roll, aerodynamic extrusion of plastic sheets, glass fiber and paper production, cooling of an infinite metallic plate in a cooling path, manufacturing of polymeric sheets are examples for practical applications of continuous moving flat surfaces. Since the pioneering work of Sakiadis [1], various aspects of the problem have been investigated by many authors. Mass transfer's analyses at the stretched sheet were enclosed in their studies by Erickson et al. [2] and relevant experimental results were reported by Tsou et al. [3] regarding several aspects for the flow and heat transfer boundary layer problems in a continuously moving sheet. Crane [4] and Gupta and Gupta [5] have analyzed the stretching problem with constant surface temperature while Soundalgekar [6] investigated the Stokes problem for a viscoelastic fluid. This flow was examined by Siddappa and Khapate [7] for a special class of non-Newtonian fluids known as second-order fluids which are viscoelastic in nature.

Rajagopal et al. [8] independently examined the same flow as in Ref. [7] and obtained similarity solutions of the boundary layer equations numerically for the case of small viscoelastic parameter k_1. It is shown that skin-friction decreases with increase in k_1. Dandapat and Gupta [9] examined the same problem with heat transfer. In Ref. [9], an exact analytical solution of the non-linear equation governing this self-similar flow which is consistent with the

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Nomenclature

A, D	prescribed constants
A_0	constant (Eqs. (23) and (29))
A^*	space-dependent internal heat generation/absorption
B^*	temperature-dependent internal heat generation/absorption
c	stretching rate
c_P	specific heat at constant pressure
E_c	Eckert number for the PST case
E'_c	scaled Eckert number for the PHF case
f	dimensionless stream function
g	dimensionless temperature for the PHF case
k	thermal conductivity
k_1	viscoelastic parameter
k_0	parameter related to N_R
k^*	mean absorption coefficient
l	characteristic length
N_R	radiation parameter
q_r	radiative heat flux
q'''	rate of internal heat generation or absorption
r	parameter related to k_1 (Eq. (13))
T	fluid temperature
u, v	velocity components along x and y directions, respectively
x, y	Cartesian coordinates along the plate and normal to it, respectively

Greek symbols

α	thermal diffusivity
α_1, α_2	normal stress moduli
η	dimensionless similarity variable
θ	dimensionless temperature for the PST case
μ	dynamic viscosity
υ	kinematic viscosity
ρ	density
σ	Prandtl number
σ^*	Stefan–Boltzmann constant

Subscripts

w, ∞	conditions at the surface and in the free stream, respectively

Superscript

$(')$	derivative with respect to η

numerical results in Ref. [8] is given and the solutions for the temperature for various values of k_1 are presented. Later, Cortell [10] extended the work of Dandapat and Gupta [9] to study the heat transfer in an incompressible second-order fluid caused by a stretching sheet with a view to examining the influence of the viscoelastic parameter on that flow. It is found that temperature distribution depends on k_1, in accordance with the results in Ref. [9].

In the case of fluids of differential type (see Ref. [11]), the equations of motion are in general one order higher than the Navier–Stokes equations and, in general, need additional boundary conditions to determine the solution completely. These important issues were studied in detail by Rajagopal [11,12] and Rajagopal and Gupta [13].

The effects of heat generation/absorption become important in view of various physical problems (see Vajravelu and Hadjinicolaou [14]) and those effects have been assumed to be constant, space-dependent or temperature-dependent (Vajravelu and Hadjinicolaou [15]). Even, very recently, the mixed convection boundary layer flow of a Newtonian, electrically conducting fluid over an inclined continuously stretching sheet with power–law temperature variation in the presence of magnetic field, internal heat generation/absorption and wall suction/injection is analyzed by Abo-Eldahab and El Aziz [16]. In the present research, we extend the problem investigated in Ref. [16] to viscoelastic fluid flows.

Furthermore, Char [17] studied MHD flow of a viscoelastic fluid over a stretching sheet, however, only the thermal diffusion is considered in the energy equation; later, Sarma and Rao [18], Vajravelu and Roper [19] and Cortell [20,21] analyzed the effects of work due to deformation in such an equation. Another effect which bears great importance on heat transfer is the viscous dissipation. The determination of the temperature distribution when the internal friction is not negligible is of utmost significance in different industrial fields, such as chemical and food processing, oil exploitation and bio-engineering. Consequently, the effects of viscous dissipation are also included in the energy equation.

On the other hand, the effect of radiation on viscoelastic boundary-layer flow and heat transfer problems can be quite significant at high operating temperature. In view of this, viscoelastic flow and heat transfer over a flat plate with constant suction, thermal radiation and without viscous dissipation were studied by Raptis and Perdikis [22]. Viscous dissipation and radiation were considered by Raptis [23] and the effect of radiation was also included in Ref. [24] and in Ref. [25]. Very recently, researches in these fields have been conducted by many investigators [26–30]; however, the effects of work due to deformation on viscoelastic flows and heat transfer in the presence of radiation, viscous dissipation and non-uniform heat source/sink have not been studied in recent years.

In the present paper a proper sign for the normal stress modulus (i.e., $\alpha_1 \geqslant 0$) is used and, as we will see in Section 3, the effects of viscous dissipation, work due to deformation, internal heat generation/absorption and thermal radiation are included in the energy equation. This last effect has been enclosed in this study by employing the Rosseland approximation [31]. Furthermore, we augment the boundary conditions to the flow problem and then, momentum and heat transfer in an incompressible and thermodynamically compatible second order fluid, which is termed as second grade fluid (see Ref. [20]), past a stretching sheet, are analyzed.

This paper runs as follows. In Section 2, we shall consider the mathematical analysis of the flow and some exact solutions of the boundary layer second grade fluid flow over a linearly stretching continuous surface; in Section 3 we shall examine the thermal problem when all the effects cited above are included in the energy equation for two cases of boundary heating: (a) prescribed surface temperature (PST case) and (b) prescribed heat flux (PHF case); furthermore, similar solutions are obtained for both stream function and temperature and the influence on the numerical results of those additional effects above-mentioned will also be discussed.

2. Flow analysis

An incompressible homogeneous fluid of second order has a constitutive equation given by [32]:

$$\mathbf{T} = -p\mathbf{I} + \mu\mathbf{A}_1 + \alpha_1\mathbf{A}_2 + \alpha_2\mathbf{A}_1^2. \tag{1}$$

Here $\mathbf{T}$ is the stress tensor, p the pressure, μ the coefficient of viscosity, α_1, α_2 are the normal stress moduli and $\mathbf{A}_1$ and $\mathbf{A}_2$ are defined as

$$\mathbf{A}_1 = (\text{grad } \mathbf{v}) + (\text{grad } \mathbf{v})^{\mathrm{T}}, \tag{2}$$

$$\mathbf{A}_2 = \mathrm{d}/\mathrm{d}t\mathbf{A}_1 + \mathbf{A}_1 \cdot \text{grad } \mathbf{v} + (\text{grad } \mathbf{v})^{\mathrm{T}} \cdot \mathbf{A}_1. \tag{3}$$

Here $\mathbf{v}$ denotes the velocity field and $\mathrm{d}/\mathrm{d}t$ is the material time derivative. Some assumptions concerning the sign of α_1 in the model (1) will be necessary. For thermodynamic reasons (see Ref. [33]), the material parameter α_1 must be positive. If the fluid of second order modelled by Eq. (1) is to be compatible with thermodynamics and is to satisfy the Clausius–Duhem inequality for all motions and the assumption that the specific Helmholtz free energy of the fluid is a minimum when it is locally at rest, then

$$\mu \geqslant 0,\ \alpha_1 \geqslant 0,\ \alpha_1 + \alpha_2 = 0.$$

The constitutive equation given by Eq. (1) is capable of modelling a non-Newtonian fluid which possesses visco-elastic ($\mu > 0$; $\alpha_1 > 0$) properties. In our analysis we assume that the fluid is thermodynamically compatible ($\alpha_1 \geqslant 0$); we consider the flow of an incompressible second grade fluid past a flat and impermeable sheet coinciding with the plane $y = 0$, the flow being confined to $y > 0$. Two equal and opposite forces are applied along the x-axis so that the wall is stretched keeping the origin fixed. The steady two-dimensional boundary layer equations for this fluid, in the usual notation, are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{4}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon\frac{\partial^2 u}{\partial y^2} + \frac{\alpha_1}{\rho}\left[\frac{\partial}{\partial x}\left(u\frac{\partial^2 u}{\partial y^2}\right) - \frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x \partial y} + v\frac{\partial^3 u}{\partial y^3}\right], \tag{5}$$

where u and v are the velocity components in x and y directions, respectively, υ is the kinematic viscosity and ρ is the density. The boundary conditions to the problem are:

$$u_{\mathrm{w}}(x) = cx,\ v = 0 \quad \text{at } y = 0,\ c > 0, \tag{6}$$

$$u \to 0,\ \frac{\partial u}{\partial y} \to 0 \quad \text{as } y \to \infty. \tag{7}$$

The second condition (7) is the augmented condition since the flow is in an unbounded domain, which has been discussed by Garg and Rajagopal [34].

Defining new variables

$$u = cxf'(\eta), \quad v = -(c \cdot v)^{1/2} f(\eta), \tag{8}$$

where

$$\eta = \left(\frac{c}{v}\right)^{1/2} y, \tag{9}$$

and substituting into Eq. (5) give

$$(f')^2 - ff'' = f''' + k_1[2f'f''' - (f'')^2 - ff^{iv}], \tag{10}$$

where $k_1 = \alpha_1 c/\rho v$ is the viscoelastic parameter and a prime denotes differentiation with respect to η. The boundary conditions (6) and (7) become

$$f = 0,\ f' = 1 \quad \text{at } \eta = 0,$$
$$f' \to 0,\ f'' \to 0 \quad \text{as } \eta \to \infty. \tag{11}$$

It is interesting to note that the problem {(10) and (11)} has a solution of the form

$$f(\eta) = (1/r) \cdot (1 - \exp(-r\eta)), \tag{12}$$

where

$$r = (1 + k_1)^{-1/2}. \tag{13}$$

This gives the velocity components

$$u = cx\exp(-r\eta),$$
$$v = -(cv)^{1/2}\frac{1 - \exp(-r\eta)}{r}. \tag{14}$$

For a purely viscous fluid (i.e., $k_1 = 0$), Eq. (10) becomes $f''' + ff'' - (f')^2 = 0$ which is satisfied by Eq. (12) with $k_1 = 0$ (i.e., $r = 1$) which is in agreement with the steady state flow's solution for a purely viscous fluid. So, for a slightly viscoelastic fluid (small k_1) we obtain from Eqs. (12) and (13) a boundary layer only slightly altered in its dimensions from the viscous one; therefore, Eqs. (12) and (13) represent a realistic solution for the flow treated here, but, in several industrially important processes, we can often encounter fast flows of highly viscoelastic fluids such as polymer melts like high-viscosity silicone oils. These real flows must be simulated in order to have some idea of how heat transfer depends on several effects which are considered in this research.

Thus, we get a simple exact analytical solution and we use in heat transfer analysis this solution for the function f.

3. Heat transfer analyses

By using usual boundary layer approximations, the equation of the energy for temperature T in the presence of radiation, with temperature dependent heat source/sink in the flow region, viscous dissipation and taking into account the work due to deformation is given by

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{v}{c_P}\left(\frac{\partial u}{\partial y}\right)^2-\frac{1}{\rho c_P}\frac{\partial q_r}{\partial y}+\frac{q'''}{\rho c_P}+\frac{\alpha_1}{\rho c_P}\frac{\partial u}{\partial y}\left[\frac{\partial}{\partial y}\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)\right], \quad (15)$$

where α is the thermal diffusivity, c_P is the specific heat of a fluid at constant pressure, q_r is the radiative heat flux and q''' is the rate of internal heat generation (>0) or absorption (<0) coefficient. The second and fifth terms that appear on the right-hand side of Eq. (15) pertain to the effects of viscous dissipation and elastic deformation, respectively.

Using the Rosseland approximation for radiation [31], the radiative heat flux is simplified as

$$q_r=-\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y}, \quad (16)$$

where σ^* and k^* are the Stefan–Boltzmann constant and the mean absorption coefficient, respectively. We assume that the temperature differences within the flow such as that the term T^4 may be expressed as a linear function of temperature. Hence, expanding T^4 in a Taylor series about T_∞ and neglecting higher-order terms we get

$$T^4\cong 4T_\infty^3 T-3T_\infty^4. \quad (17)$$

In view of Eqs. (16) and (17), Eq. (15) reduces to

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\left(\alpha+\frac{16\sigma^* T_\infty^3}{3\rho c_P k^*}\right)\frac{\partial^2 T}{\partial y^2}+\frac{v}{c_P}\left(\frac{\partial u}{\partial y}\right)^2+\frac{q'''}{\rho c_P}+\frac{\alpha_1}{\rho c_P}\frac{\partial u}{\partial y}\left[\frac{\partial}{\partial y}\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)\right]. \quad (18)$$

From the above equation it is seen that the effect of radiation is to enhance the thermal diffusivity. The term q''' is modelled as:

$$q'''=\frac{ku_w(x)}{x\upsilon}(A^*(T_w-T_\infty)f'(\eta)+B^*(T-T_\infty)), \quad (19)$$

where T_w is the temperature at the wall, T_∞ is the fluid temperature far away from the surface, k is the thermal conductivity and A^* and B^* are the parameters of space-dependent and temperature-dependent internal heat generation (i.e., $A^*>0$ and $B^*>0$)/absorption (i.e., $A^*<0$ and $B^*<0$), respectively. Two kinds of thermal boundary condition at the wall are considered and they are treated separately in the following sections.

3.1. Prescribed surface temperature (PST case)

In this circumstance, the boundary conditions are

$$T=T_w\left(=T_\infty+A\cdot\left(\frac{x}{l}\right)^2\right) \quad \text{at } y=0, \qquad T\to T_\infty \quad \text{as } y\to\infty, \quad (20)$$

where the constant l is chosen as a characteristic length.

On the other hand, we define the non-dimensional temperature $\theta(\eta)$ as:

$$\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}. \quad (21)$$

Realize that in order to obtain similarity solutions for temperature $\theta(\eta)$ we consider stretched boundary surface with prescribed power law temperature of second grade only (see Ref. [17]).

Using Eqs. (8), (9), (19), (20) and (21) we find from (18)

$$\theta''-\frac{3\sigma N_R}{3N_R+4}(2f'\theta-f\theta')+\frac{3N_R}{3N_R+4}(A^*f'+B^*\theta)=-\frac{3\sigma N_R}{3N_R+4}E_c[(f'')^2+k_1f''(f'f''-ff''')]. \quad (22)$$

Here, $\sigma=\frac{\upsilon}{\alpha}$ is the Prandtl number, $E_c=\frac{c^2l^2}{Ac_P}$ is the Eckert number and $N_R=\frac{kk^*}{4\sigma^* T_\infty^3}$ is the radiation parameter.

In view of Eq. (12), Eq. (22) reduces to

$$\theta''-\sigma k_0\left[2\theta\exp(-r\eta)-\frac{1-\exp(-r\eta)}{r}\theta'\right]+k_0(A^*\exp(-r\eta)+B^*\theta)=-\sigma k_0E_cA_0r^2\exp(-2r\eta), \quad (23)$$

where $k_0=\frac{3N_R}{3N_R+4}$; $k_0=1$ with and without thermal radiation, respectively and $A_0=1+k_1$.

It is clear from the above equation that the effect of the work due to deformation is to enhance the dissipative term. Eq. (23) also governs a set of particular problems. For example, with $A^*\neq 0$ and $B^*\neq 0$, we can consider only one of the following three effects, dissipative heat (i.e., $k_0=1$; $A_0=1$); elastic deformation (i.e., $k_0=1$; $A_0=k_1$) or thermal radiation (i.e., $k_0=\frac{3N_R}{3N_R+4}$; $A_0=0$).

The boundary conditions (20) become

$$\theta(0)=1,\theta(\infty)\to 0. \quad (24)$$

Taking into account the thermal radiation, we can express the surface heat flux as:

$$q_w=-k\left(\frac{\partial T}{\partial y}\right)_w+(q_r)_w=-\frac{k}{k_0}A\left(\frac{c}{\upsilon}\right)^{1/2}\left(\frac{x}{l}\right)^2\theta'(0). \quad (25)$$

Using numerical methods of integration and disregarding temporarily the second condition (24), a family of solutions of (23) can be obtained for arbitrarily chosen values of $\left(\frac{d\theta}{d\eta}\right)_{\eta=0}=\theta'(0)\leqslant 0$. Tentatively we assume that a special value of $|\theta'(0)|$ yields a solution for which θ vanishes at a certain $\eta=\eta_\infty$ and satisfies the additional condition

$$\frac{d\theta}{d\eta} = 0, \ \theta = 0 \quad \text{at } \eta = \eta_\infty. \tag{26}$$

We guess $\theta'(0)$ and integrate Eq. (23) together with first condition (24) as an initial value problem by the Runge–Kutta method of fourth order with the additional condition (26). We follow an iterative procedure which is stopped to give the temperature profiles when (26) is reached. In the present analysis the equivalent step size $\Delta\eta = 0.02$ is used to obtain the numerical solution. It is worth mentioning that, for each numerical solution, the η_∞ value depends on the non-dimensional parameters which govern energy and momentum boundary layer problems. As we will see, this paper highlights the effects of elastic deformation and frictional heating on temperature distributions taking into account the presence of a non-uniform heat source and thermal radiation.

3.2. Prescribed heat flux (PHF case)

In this case, the power–law heat flux on the wall is considered in the form

$$\text{at } y = 0 : q_w = -k\left(\frac{\partial T}{\partial y}\right)_w = D\left(\frac{x}{l}\right)^2,$$
$$\text{as } y \to \infty : T \to T_\infty. \tag{27}$$

where D is a constant.

On the other hand, we define a non-dimensional temperature $g(\eta)$ as

$$g(\eta) = \frac{T - T_\infty}{\frac{D}{k}\left(\frac{x}{l}\right)^2\left(\frac{\upsilon}{c}\right)^{1/2}}. \tag{28}$$

Using Eqs. (12), (19) and (28), we find from the energy equation (18):

$$g'' - \sigma k_0\left[2g\exp(-r\eta) - \frac{1-\exp(-r\eta)}{r}g'\right] + k_0(A^*g(0)\exp(-r\eta) + B^*g) = -\sigma k_0 E_c' A_0 r^2 \exp(-2r\eta). \tag{29}$$

where $k_0 = \frac{3N_R}{3N_R+4}$; $k_0 = 1$ with and without thermal radiation, respectively and $E_c' = \frac{E_c A k}{D}\left(\frac{c}{\upsilon}\right)^{1/2}$ is the scaled Eckert number.

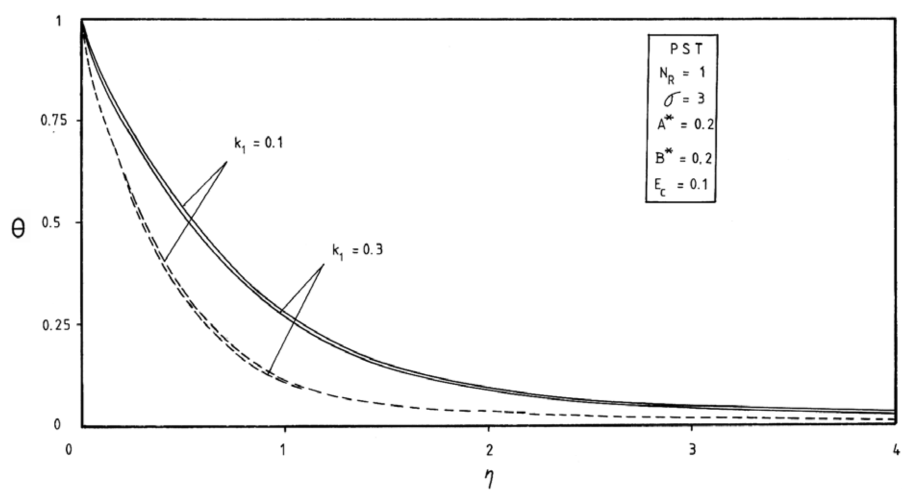

Fig. 1a. Temperature profiles in PST case for two values of k_1 with (solid line) and without (broken line) thermal radiation.

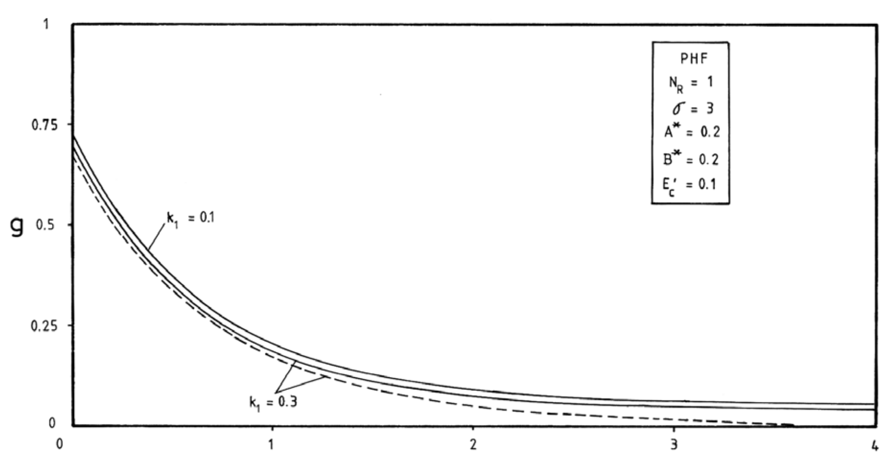

Fig. 1b. Temperature profiles in PHF case for two values of k_1 with (solid line) and without (broken line) work due to deformation.

The boundary conditions can be obtained from Eqs. (27) and (28) as

$$g'(0) = -1; \quad g(\infty) = 0 \tag{30}$$

and in view of Eq. (28), we get

$$T_w = T_\infty + \frac{D}{k}\left(\frac{x}{l}\right)^2\left(\frac{\upsilon}{c}\right)^{1/2} g(0). \tag{31}$$

Temperature profiles with and without different effects in the PST/PHF cases are depicted in Figs. 1–6 for several values of the parameters $E_c(E'_c)$, k_1, N_R, σ, A^* and B^*. The effect of increasing k_1 is shown in Figs. 1a and 1b for both PST and PHF cases. It is clear from these Figures that as k_1 increases, a slight decrease in temperature occurs. Fig. 1a elucidates the influence of the thermal radiation's effect on temperature distribution for PST case, whereas Fig. 1b (PHF case) depicts the variation of temperature profiles for two values of k_1 when the elastic deformation's effect is present or it is absent. As we can see in Fig. 1b, such an effect becomes more important when k_1 is high.

Figs. 2a and 2b depict the effect of varying $E_c(E'_c)$ for $N_R = 1$; $\sigma = 3$; $A^* = 0.2$; $B^* = 0.2$ and $k_1 = 0.1$. The results show marked increase in the temperature distributions with increase in $E_c(E'_c)$ for both PST and PHF cases. As we can see, internal heat generation and work done by deformation yield an augment in the fluid's temperature and these effects increase with increase in $E_c(E'_c)$.

Figs. 3a and 3b depict the effect of varying N_R for $\sigma = 3$; $A^* = 0.2$; $B^* = 0.2$; $k_1 = 0.1$ and $E_c(E'_c) = 0.1$. In general, the results show marked decrease in the temperature distributions with increase in N_R for both PST/PHF cases. It is obvious that dissipative heat and, again, the work due to deformation yield an augment in the fluid's temperature. Realize that the effect of fluid's elasticity becomes more important when N_R is low (see Fig. 3b).

The effect of increasing σ is shown in Figs. 4a and 4b for both PST/PHF cases when $N_R = 1$; $A^* = 0.2$; $B^* = 0.2$; $k_1 = 0.1$ and $E_c(E'_c) = 0.1$. The results show marked decrease in temperature profiles with increase in σ for all the cases. Moreover, the effect of the dissipative heat on temperature distributions has also been depicted. Again,

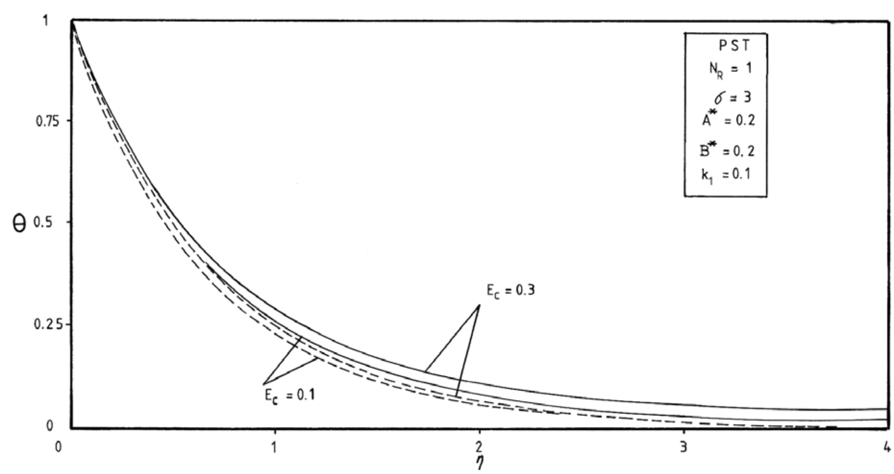

Fig. 2a. Temperature profiles in PST case for two values of E_c with (solid line) and without ($A^* = B^* = 0$, broken line) internal heat generation.

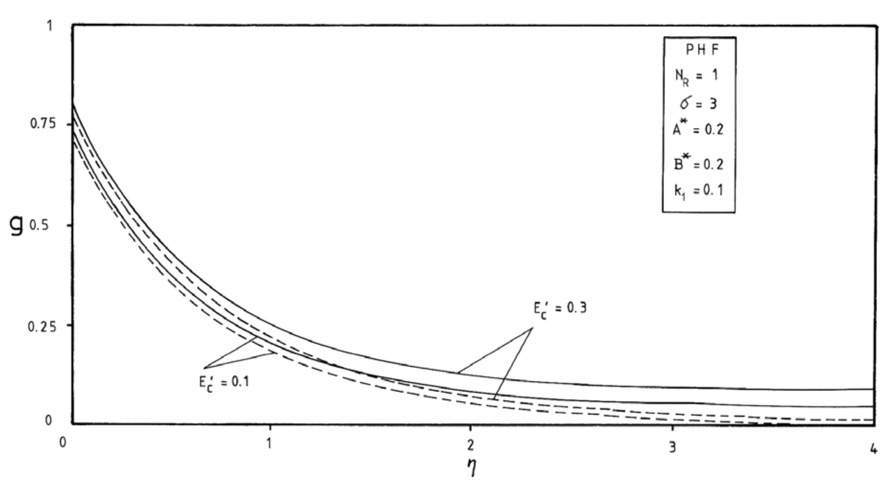

Fig. 2b. Temperature profiles in PHF case for two values of E'_c with (solid line) and without (broken line) work done by deformation.

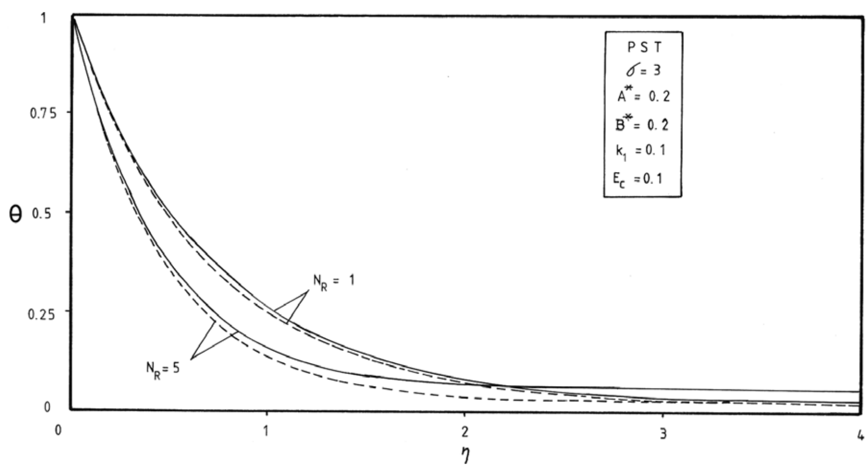

Fig. 3a. Temperature profiles in PST case for two values of N_R with (solid line) and without (broken line) viscous dissipation.

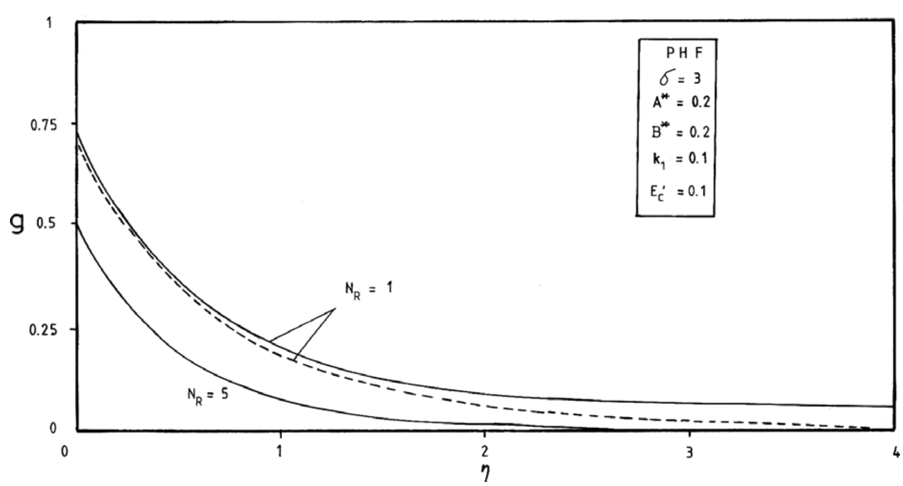

Fig. 3b. Temperature profiles in PHF case for two values of N_R with (solid line) and without (broken line) work done by deformation.

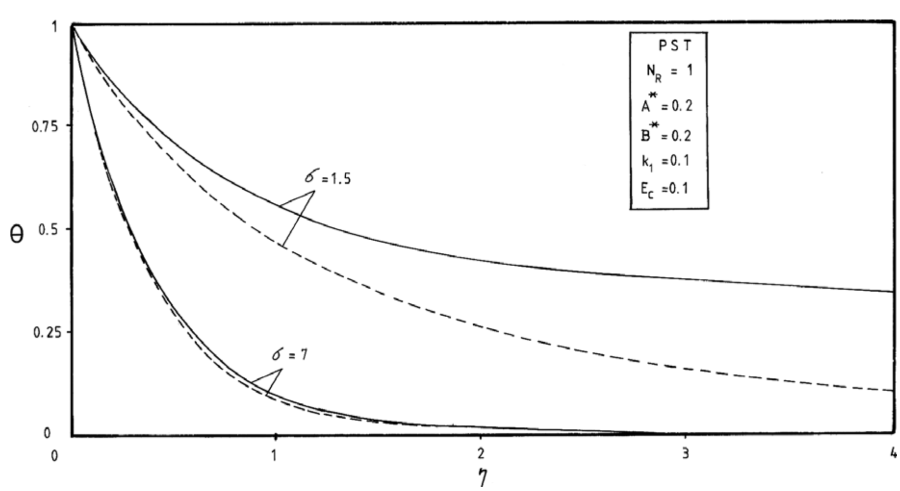

Fig. 4a. Temperature profiles in PST case for two values of σ when the dissipative heat is present (solid line) or it is absent (broken line).

when the effects of frictional heating are considered in the energy equation, an augment in the fluid's temperature occurs. These augments become more important when σ is low.

The effect of increasing A^* is shown in Figs. 5a and 5b for both PST/PHF cases when $N_R = 1$; $\sigma = 3$; $B^* = 0.2$; $k_1 = 0.1$ and $E_c(E'_c) = 0.1$. As seen, the results indicate slight increase in temperature profiles with increase in A^*

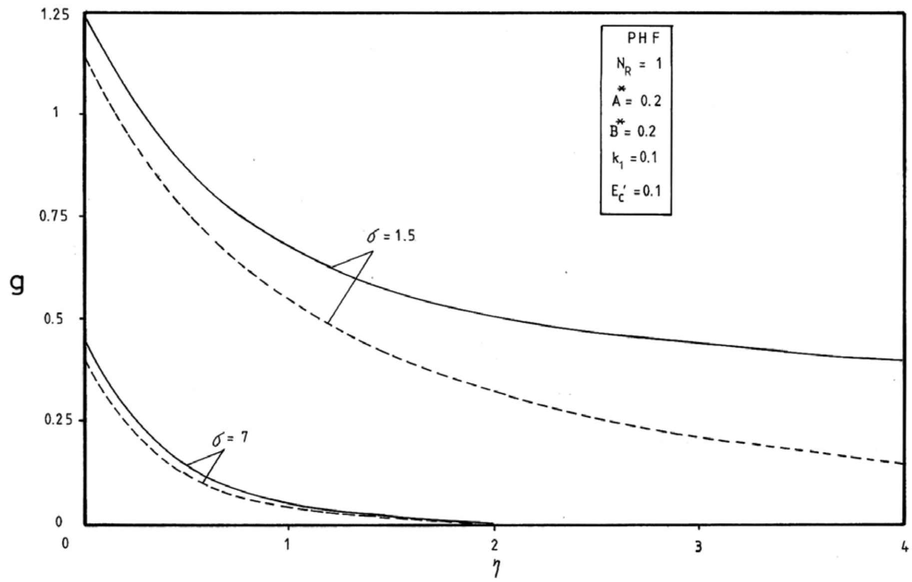

Fig. 4b. Temperature profiles in PHF case for two values of σ when the dissipative heat is present (solid line) or it is absent (broken line).

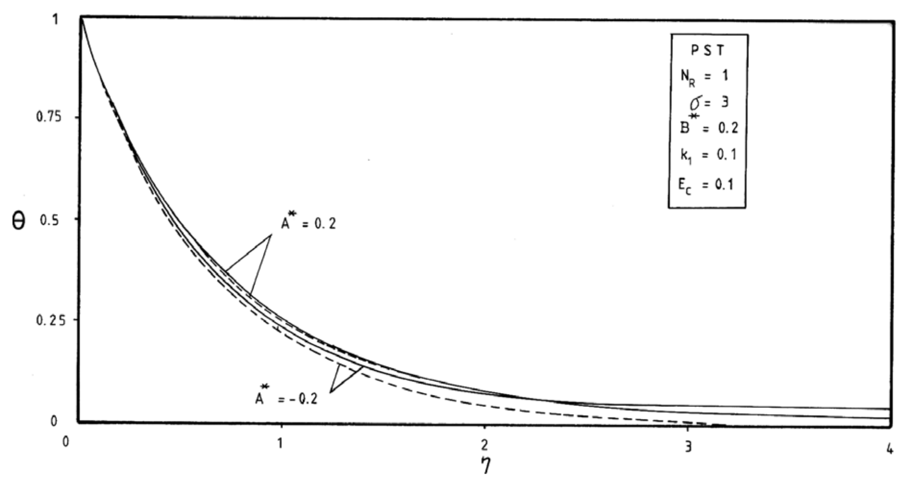

Fig. 5a. Temperature profiles in PST case for two values of A^* with (solid line) and without (broken line) elastic deformation's effect.

for all the cases. The frictional heating yields an augment in the temperature of the fluid in the flow region and the elastic deformation's effect, too.

Figs. 6a and 6b depict the effect of varying B^* for $N_R = 1$; $\sigma = 3$; $A^* = 0.2$; $k_1 = 0.1$ and $E_c(E_c') = 0.1$. The results indicate slight increase in temperature distributions with increase in B^* for both PST/PHF cases.

The combined effect of increasing values of σ and N_R is to decrease the magnitude of $\theta(\eta)$ largely in the boundary layer flow region. The effect of increasing $E_c(E_c')$ is to enhance the temperature $\theta(\eta)$, whereas the effect of increasing k_1 is quite the opposite. The internal heat generation/absorption enhances or damps the heat transport. Based on numerical results treated here, the influence of the elastic deformation on temperature profiles decreases when σ and N_R increase whereas it increases when k_1 and $E_c(E_c')$ increase.

Finally, the values of the wall temperature gradient $[-\theta'(0)]$ and the wall temperature $g(0)$ as a function of all the parameters of the thermal boundary-layer treated here, have been tabulated in Table 1. From this Table we observe that the effect of viscoelastic parameter k_1 is to increase the wall temperature gradient $[-\theta'(0)]$ in PST case and to decrease the wall temperature $g(0)$ in PHF case. The effect of increasing $E_c(E_c')$ is to increase the magnitude of both $\theta(\eta)$ and $g(\eta)$, whereas the opposite behaviour is seen for both the parameters N_R and σ. The effect of increasing the strength of the heat sink is to increase the wall temperature gradient $[-\theta'(0)]$ and the opposite behaviour is seen for a heat source.

4. Discussion and conclusions

In this work we analyze boundary-layer flow and heat transfer in a viscoelastic fluid over a stretching sheet in the presence of radiation and the Rosseland approximation for the radiative heat flux is used. A parameter of interest for the present study is the viscoelastic parameter k_1 which

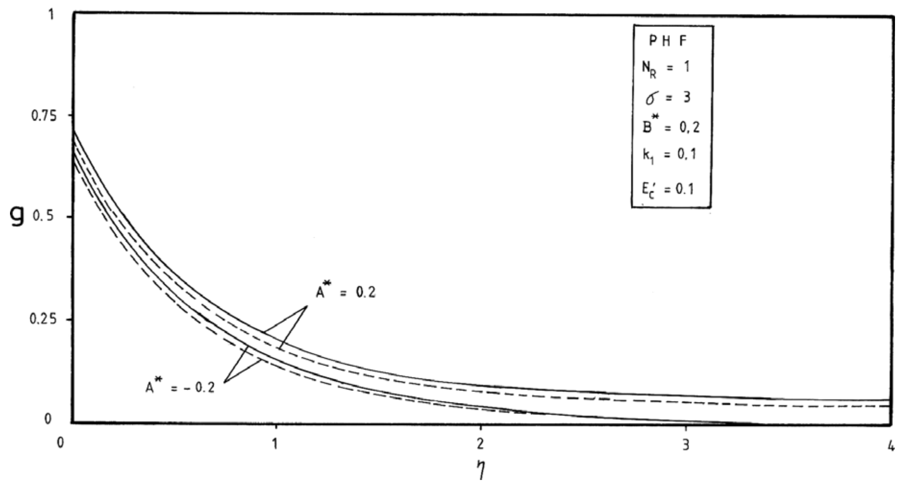

Fig. 5b. Temperature profiles in PHF case for two values of A^* when the dissipative heat is present (solid line) or it is absent (broken line).

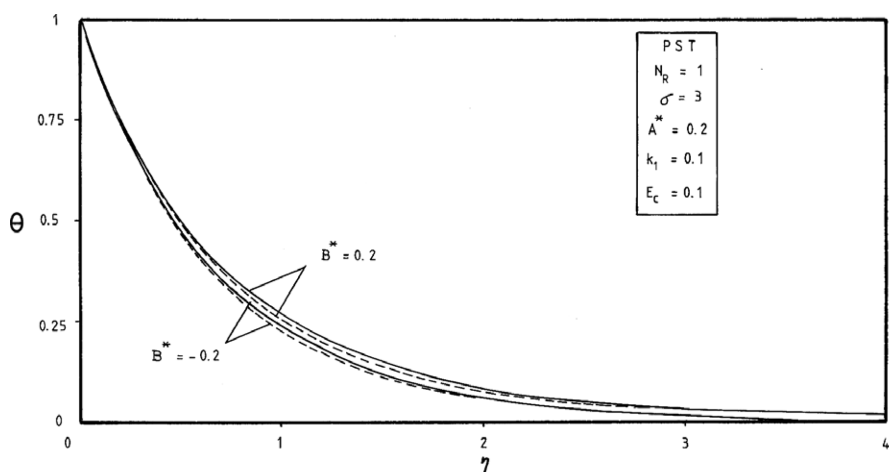

Fig. 6a. Temperature profiles in PST case for two values of B^* when the dissipative heat is present (solid line) or it is absent (broken line).

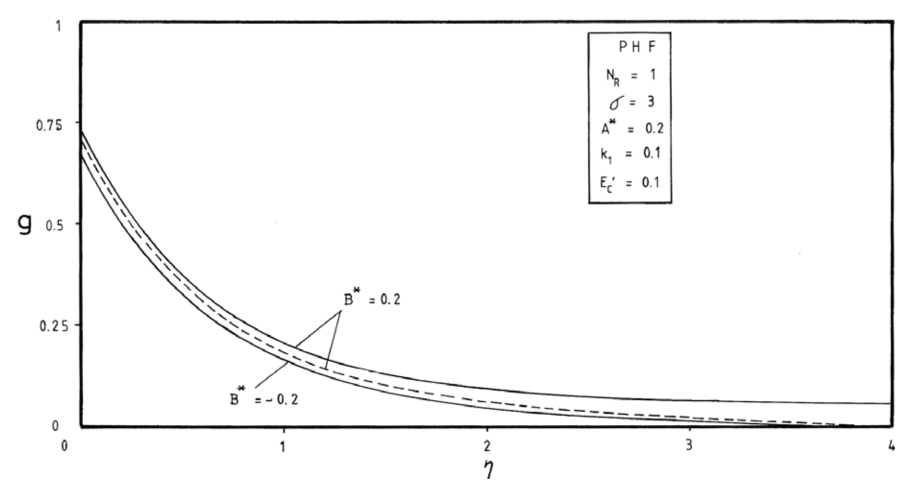

Fig. 6b. Temperature profiles in PHF case for two values of B^* with (solid line) and without (broken line) elastic deformation's effect.

is related to α_1. The values of f' and f are related to the velocity components u and v through Eqs. (12)–(14). From these equations it can be studied the behaviour of u and v with changes in k_1. In view of this, the velocity components

Table 1
Wall temperature gradient $[-\theta'(0)]$ (PST case) and wall temperature $g(0)$ (PHF case) for several values of k_1, $E_c(E'_c)$, N_R, σ, A^* and B^*

k_1	$E_c(E'_c)$	N_R	σ	A^*	B^*	$-\theta'(0)$	$g(0)$
0.0	0.1	1	3	0.2	0.2	1.41679	0.71619
0.1						1.43867	0.71265
0.3						1.45673	0.70168
0.1	0.0	1	3	0.2	0.2	1.46377	0.68768
	0.1					1.43867	0.71265
	0.3					1.34648	0.77721
0.1	0.1	1	3	0.2	0.2	1.43867	0.71265
		3				1.88649	0.54324
		5				2.02060	0.50689
0.1	0.1	1	1.5	0.2	0.2	0.80103	1.23417
			3			1.43867	0.71265
			7			2.02060	0.50689
0.1	0.1	1	3	−0.2	0.2	1.51686	0.66227
				0.0		1.47677	0.69764
				0.2		1.43867	0.71265
0.1	0.1	1	3	0.2	−0.2	1.51037	0.67104
					0.0	1.47650	0.68608
					0.2	1.43867	0.71265

above-mentioned are a decreasing function of η. We also observe that r is an important parameter in the present study which is related to k_1 through Eq. (13).

The equations for the heat transfer analysis were solved by the Runge–Kutta method of fourth order and the influences of the parameters $E_c(E'_c)$, k_1, N_R, σ, A^* and B^* on temperature profiles were examined in this analysis.

From our numerical results and for both PST/PHF cases, the following conclusions may be drawn:

1. The increase of the parameter $E_c(E'_c)$ leads to the increase of dimensionless surface temperature.
2. An augment in k_1 yields a diminution in the temperature of the fluid.
3. The combined effect of increasing values of σ and N_R is to decrease the temperature distribution in the flow region with its increases.
4. The effect of the inclusion of viscous dissipation is to increase the temperature distribution in the flow region.
5. The internal heat generation/absorption enhances or damps the heat transport.
6. The influence of the work due to deformation on temperature profiles becomes more significant when the parameters σ and N_R are low.
7. The presence of the contribution of heat due to elastic deformation in the energy equation yields an augment in the temperature of the fluid.

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